Math Ma

Logarithm Rules

Fall 2016

1. Evaluate the following. (Your answer would not involve logarithm!)

(a) $\log_2(2^3 \cdot 2^4)$

(b) $\log \dfrac{10^2}{10^5}$

(c) $\ln\left(e^{-\frac{1}{2}}\right)$

(d) $\log_{\sqrt{3}} 3$

2. Let $x = \ln A, y = \ln B$ and p is a number. Express the following only in terms of x, y and p.

(a) $\ln(A \cdot B)$

(b) $\ln \dfrac{A}{B}$

(c) $\ln A^p$

(d) $\log_{e^p} A$

3. Using what we observed in Problem 2, evaluate the following.

(a) $\log_2 5 + \log_2 0.8$

(b) $\log 2.5 - \log 25$

(c) $\ln \dfrac{1}{\sqrt{e}}$

(d) $\log_{\frac{1}{9}} 3$

Rule for Exponentials	**Rule for Logarithms**
$b^x b^y = b^{x+y}$	$\log_b A \cdot B = \log_b A + \log_b B$
$\dfrac{b^x}{b^y} =$	$\log_b \dfrac{A}{B} =$
$(b^x)^p =$	$\log_b A^p =$
	$\log_{b^p} A =$

4. Solve x for each of the following equation.

(a) $2^{2x+3} = 5$

(b) $5^{x^2+4x-1} = 25^{x-1}$

5. Solve the equation $5^x = 17$ in two ways. One by taking $\log_5$ of both sides and the other by taking ln of both sides.

First method:

$$5^x = 17$$
$$\log_5(5^x) = \log_5 17$$
$$x \stackrel{?}{=}$$

Second method:

$$5^x = 17$$
$$\ln 5^x = \ln 17$$
$$x \stackrel{?}{=}$$

Change of base for logarithm

$$\log_b x = \frac{\ln x}{\ln b}$$

6. (a) $\log_5 7 \cdot \log_7 5$

(b) $\left(e^{\log_5 2}\right)^{\ln 5}$

(c) $\log_2 9 + \log_4 3$

(d) $\log_2 9 \cdot \log_3 4$

7. Simplilfy each of the following as much as possible. **Two** of them will still have logarithm in the final answer.

(a) $\ln \frac{e^2}{4} + 2 \ln 2$

(b) $\log_2 \sqrt{\frac{3}{8}} - \log_2 3 + \log_4 3$

(c) $\log_7(7^2 + 7^3)$

(d) $\log_3 5 \cdot \log_3 25$

(e) $2^{\log_4 5}$

(f) $5^{\log \frac{2}{5}} \cdot 25^{\log 5}$

Logarithm Rules – Solutions

1. (a) $\log_2(2^3 \cdot 2^4) = \log_2 2^7 = 7$
   (b) $\log \dfrac{10^2}{10^5} = \log 10^{-3} = -3$
   (c) $\ln\left(e^{-\frac{1}{2}}\right) = -\dfrac{1}{2}$
   (d) $\log_{\sqrt{3}} 3 = 2$

2. In exponential language we have $A = e^x, B = e^y$.
   (a) We have $A \cdot B = e^{x+y}$, so $\ln(A \cdot B) = x + y$.
   (b) We have $\dfrac{A}{B} = e^{x-y}$, so $\ln \frac{A}{B} = x - y$.
   (c) We have $A^p = (e^x)^p = e^{px}$, so $\ln A^p = px$.
   (d) We have $A = e^x = (e^p)^{\frac{x}{p}}$, so $\log_{e^p} A = \frac{x}{p}$.

3. (a) $\log_2 5 + \log_2 0.8 = \log_2(5 \cdot 0.8) = \log_2 4 = 2$.
   (b) $\log 2.5 - \log 25 = \log \dfrac{2.5}{25} = \log \dfrac{1}{10} = -1$.
   (c) $\ln \dfrac{1}{\sqrt{e}} = \ln e^{-\frac{1}{2}} = -\dfrac{1}{2}$.
   (d) $\log_{\frac{1}{9}} 3 = \log_{3^{-2}} 3 = -\frac{1}{2}$.

4. (a) There are two ways to solve the equation. First of all, we can use exponential rule to simplify the equation

$$\begin{aligned} 2^{2x+3} &= 5 \\ 2^{2x} &= \frac{5}{8} \\ 4^x &= \frac{5}{8} \\ x &= \log_4 \frac{5}{8} \end{aligned}$$

   The other method is to directly take $\log_2$ on both sides. By which we have

$$2x + 3 = \log_2 5$$

   and hence

$$x = \frac{1}{2}(\log 25 - 3)$$

   (b) To solve $5^{x^2+4x-1} = 25^{x-1}$, we first make everything base 5.

$$5^{x^2+4x-1} = 5^{2x-2}.$$

Then the exponent must be the same, so we arrive at a quadratic equation

$$\begin{aligned} x^2+4x-1&=2x-2\\ x^2+2x+1&=0\\ (x+1)^2&=0\\ x&=-1 \end{aligned}$$

5.

6. (a) $\log_5 7\cdot\log_7 5=\dfrac{\ln 7}{\ln 5}\cdot\dfrac{\ln 5}{\ln 7}=1$

   (b) $\left(e^{\log_5 2}\right)^{\ln 5}=e^{\frac{\ln 2}{\ln 5}\cdot\ln 5}=e^{\ln 2}=2$

   (c) $\log_2 9+\log_4 3=\log_2 3^2+\log_{2^2} 3=2\log_2 3+\dfrac{1}{2}\log_2 3=\dfrac{5}{2}\log_2 3$

   (d) $\log_2 9\cdot\log_3 4=\dfrac{\ln 9}{\ln 2}\cdot\dfrac{\ln 4}{\ln 3}=\dfrac{2\ln 3}{\ln 2}\cdot\dfrac{2\ln 2}{\ln 3}=4$

7. (a) $\ln\dfrac{e^2}{4}+2\ln 2=\ln e^2-\ln 4+2\ln 2=2-2\ln 2+2\ln 2=2$

   (b) $\log_2\sqrt{\dfrac{3}{8}}-\log_2 3+\log_4 3=\dfrac{1}{2}(\log_2 3-\log_2 8)-\log_2 3+\log_{2^2} 3=\dfrac{1}{2}\log_2 3-\dfrac{3}{2}-\log_2 3+\dfrac{1}{2}\log_2 3=-\dfrac{3}{2}$

   (c) $\log_7(7^2+7^3)=\log_7 7^2(1+7)=\log_7 7^2+\log_7 8=2+\log_7 8$

   (d) $\log_3 5\cdot\log_3 25=\log_3 5\cdot 2\log_3 5=2(\log_3 5)^2$

   (e) $2^{\log_4 5}=2^{\frac{1}{2}\log_2 5}=\left(2^{\log_2 5}\right)^{\frac{1}{2}}=5^{\frac{1}{2}}=\sqrt{5}$

   (f) $5^{\log\frac{2}{5}}\cdot 25^{\log 5}=5^{\log\frac{2}{5}+2\log 5}=5^{\log 2-\log 5+2\log 5}=5^{\log 2+\log 5}=5^{\log 10}=5$