Math Ma

Logarithms

Fall 2016

1. Let $f(x) = 2^x$.

   (a) Is $f(x)$ an invertible function? How do you know?

   (b) What is the meaning of $f^{-1}(x)$.

   (c) Evaluate the following:

   $\log_2 32 =$ _____. $\log_2 \frac{1}{4} =$ _____. $\log_2 \sqrt{2} =$ _____. $\log_2 1 =$ _____.

   (d) $2^{\log_2 x} =$ __________. $\log_2 2^x =$ __________.

   (e) What are the domain and range of $\log_2 x$?

   (f) Sketch the graph of $f(x) = 2^x$ and $f^{-1}(x) = \log_2 x$ on the same set of axis.

2. Use the fact that $\frac{d}{dx}e^{kx} = ke^{kx}$ to calculate $\frac{d}{dx}2^x$.

3. Simplify each of the following as much as possible.

(a) $\log_5 25$

(b) $\log \frac{1}{10\sqrt{10}}$

(c) $\ln e^{\pi}$

(d) $e^{-3\ln 2}$

(e) $7^{\log_7 3}$

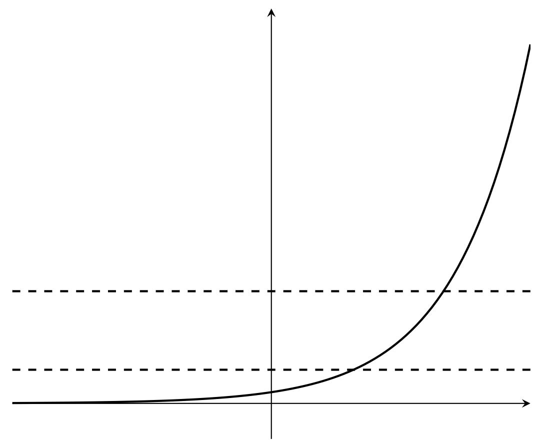
(f) $11^{\log_5 11}$

(g) $\log_3 (3^4 \cdot 3^7)$

(h) $\log_2 \left(\frac{2^6}{2^{11}}\right)$

Logarithms – Solutions

1. (a) $f(x) = 2^x$ is invertible because it passes through the horizontal line test.



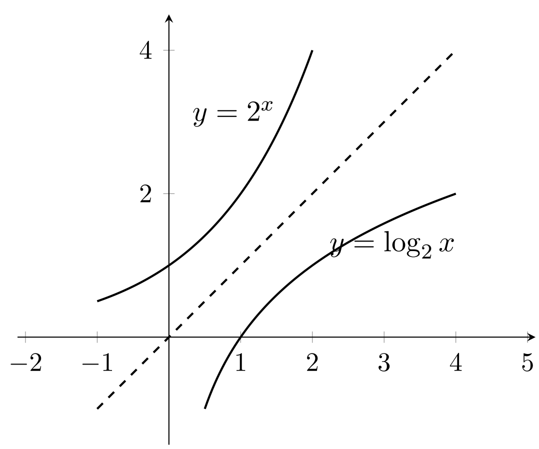
   (b) $f^{-1}(x)$ is the power we have to raise 2 to to get x.

   (c) $\log_2 32 = 5.\log_2 \dfrac{1}{4} = -2.$ $\log_2 \sqrt{2} = \dfrac{1}{2}.$ $\log_2 1 = 0.$

   (d) $2^{\log_2 x} = x.$ $\log_2 2^x = x.$ This is by definition of the inverse function $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x.$

   (e) The domain of $\log_2 x$ is the range of 2^x, which is $(0, \infty)$. The range of $\log_2 x$ is the domain of 2^x, which is $(-\infty, \infty)$.

   (f)



2.

$$\frac{d}{dx} 2^x = \frac{d}{dx} e^{\ln 2 \cdot x} = \ln 2 \cdot e^{\ln 2 \cdot x} = \ln 2 \cdot 2^x$$

3. (a) $\log_5 25 = 2$

   (b) $\log \dfrac{1}{10\sqrt{10}} = \dfrac{3}{2}$

(c) $\ln e^{\pi} = \pi$

(d) $e^{-3\ln 2} = (e^{\ln 2})^3 = 2^3 = 8$

(e) $7^{\log_7 3} = 3$

(f) $11^{\log_5 11}$

(g) $\log_3(3^4 \cdot 3^7) = \log_3(3^{11}) = 11$

(h) $\log_2 \left(\dfrac{2^6}{2^{11}} \right) = \log_2 2^{-5} = -5$