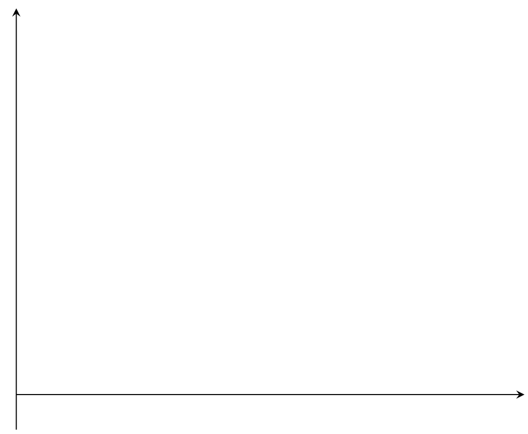
Math Ma **Inverse Functions** Fall 2016

1. Last weekend, Tegan went for a long leisurely walk around Boston. Here is the graph of $f(t)$, her distance in miles $t$ hours after the start of her walk.

(a) How far has Tegan walked after $t$ hours?

| time $t$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| distance $x$ | | | | | | |

(b) How long did it take for Tegan to walk $x$ miles?

| distance $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| time $t$ | | | | | |

(c) What does the function $f^{-1}$, the inverse function of $f$, represent?

(d) How does the domain and range of $f^{-1}$ relates to those of $f$?

(e) What is $f^{-1}(f(t))$? How about $f(f^{-1}(x))$?

(f) Sketch a graph of $f^{-1}(x)$ above. What does the two axes mean?

2. (a) Let $f(x) = x^3$.
Is $f(x)$ invertible? If it is, what is $f^{-1}(x)$? If not, how can we restrict the domain to make it invertible?

Sketch $f(x)$ and $f^{-1}(x)$ on the same set of axes.

(b) Let $g(x) = x^2$.
Is $g(x)$ invertible? If it is, what is $g^{-1}(x)$? If not, how can we restrict the domain to make it invertible?

Sketch $g(x)$ and $g^{-1}(x)$ on the same set of axes.

3. Is $f(x)^{-1}$ the same as $f^{-1}(x)$?

4. (a) $f(x) = 2^x - 1$.

(b) $g(x) = -2x^6 + 7x^5 - \pi x + \sqrt{3}$.

(c) $h(x) = x^3 + x$.

(d) $r(x) = \dfrac{1}{x-2}$

(e) $s(x) = \sqrt{x+5}$

You can actually find the inverse function of **two** of the above functions. Which ones, and what are their inverses?

# Inverse Functions – Solutions

1. (a)

| time $t$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| distance $x$ | 1.2 | 2 | 2.1 | 2.5 | 3 | 5 |

(b)

| distance $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| time $t$ | 0.8 | 2 | 5 | 5.5 | 6 |

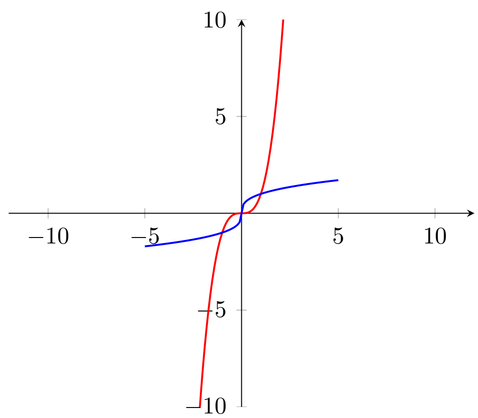
(c) $f^{-1}(x)$ is the number of hours it takes for Tegan to walk $x$ miles.

(d) The domain of $f^{-1}$ is the range of $f$ and the range of $f$ is the domain of $f$.

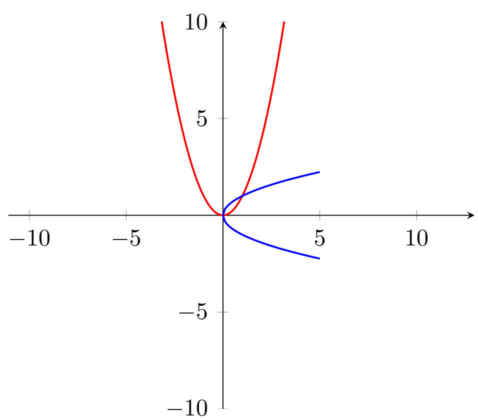
(e) $f^{-1}(f(t)) = t$. $f(f^{-1}(x)) = x$.

(f)

2. (a) $f(x)$ is invertible and $f^{-1}(x) = \sqrt[3]{x}$.




(b) $g(x)$ is not invertible. If we restrict the domain to $[0, \infty)$, then $g^{-1}(x) = \sqrt{x}$. Or if we restrict the domain to $(-\infty, 0]$, then $g^{-1}(x) = -\sqrt{x}$.



3. $f(x)^{-1}$ is not the same as $f^{-1}(x)$. For example when $f(x) = x$, then $f^{-1}(x) = x$ and $f(x)^{-1} = \frac{1}{x}$.

4. (a) $f$ is always increasing, so it is invertible. The domain of $f^{-1}$ is the range of $f$, which is $(-1, \infty)$.

   (b) Since

   $$\lim_{x\to\infty} g(x) = \lim_{x\to-\infty} g(x) = -\infty,$$

   $g(x)$ cannot be one-to-one, and thus it is not invertible.

   (c) The derivative $h'(x) = x^2 + 1 > 0$, so $h(x)$ is always increasing and thus is invertible. The domain of $h^{-1}$ is $(-\infty, \infty)$.

   (d) $r(x)$ is invertible and $r^{-1}(x) = \frac{1}{x} + 2$. The domain of $r^{-1}(x)$ is $(-\infty, 0) \cup (0, \infty)$.

   (e) $s(x)$ is invertible and $s^{-1}(x) = x^2 - 5$. The domain of $s^{-1}(x)$ is $[0, \infty)$.