Math Ma

Derivative of Exponential Functions

Fall 2016

1. Let $f(x) = 2^x$.

   (a) Use the limit definition to find $f'(x)$.

   (b) Sketch the graph of $f(x)$ and $f'(x)$.

2.

$f(x) = 2^x$

x	$f(x)$	estimate of $f'(x)$
0	1	0.693
1	2	1.386
2	4	2.774
3	8	5.547
4	16	11.094

$g(x) = 3^x$

x	$g(x)$	estimate of $g'(x)$
0	1	1.099
1	3	3.298
2	9	9.893
3	27	29.679
4	81	89.020

$h(x) = 10^x$

x	$h(x)$	estimate of $h'(x)$
0	1	2.305
1	10	23.052
2	100	230.524
3	1000	2305.238
4	10,000	23052.381

What do you observe from the table?

3. What is the number e? Roughly how big is it?

4. If k is any constant, what is the derivative of e^{kx}?

5. Find the derivative of the following functions. **You do not need quotient rule!**

(a) $f(t) = e^t + e^\pi + \pi^e$

(b) $f(t) = \dfrac{1}{7e^{2t}}$

(c) $f(t) = \sqrt{e^t}$

(d) $f(t) = \dfrac{e^t + e^{4t}}{e^{2t}}$

(e) $f(t) = (t^2 - 2t + 5)e^{4t}$

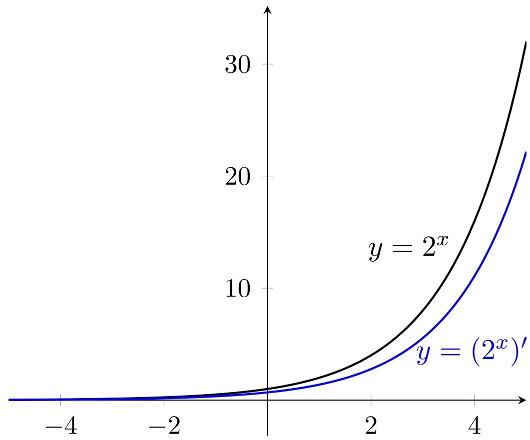
(f) $f(t) = e^2 + e + 1$

6. Use the tangent line approximation of e^x to approximate $e^{0.1}$.

Derivative of Exponential Functions – Solutions

1. (a)
$$f'(x) = \lim_{h\to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h\to 0} \frac{2^x(2^h - 1)}{h} = 2^x \cdot \lim_{h\to 0} \frac{2^h - 1}{h}$$

   (b)



2. The derivative of an exponential function is proportional to the original function. The constant of proportionality are about 0.693, 1.099 and 2.305 for base 2, 3 and 10, respectively.

3. The number e is defined to be the number such that the tangent line at $x = 1$ of the exponential function e^x has slope 1. e is between 2 and 3.

4. The derivative of e^{kx} is ke^{kx}.

5. (a) e^t
   (b) $-\frac{2}{7}e^{-2t}$
   (c) $\frac{1}{2}e^{0.5t}$
   (d) $-e^t + 2e^{2t}$
   (e) $(2t-2)e^{4t} + 4(t^2 - 2t + 5)e^{4t}$
   (f) 0

6. The tangent line of e^x at $x = 0$ passes through $(0, 1)$ and has slope 1. Hence the equation of the tangent line is
$$y - 1 = x.$$
We use the tangent line to approximate $e^{0.1} \approx 0.1 + 1 = 1.1$. This is an underestimate as exponential functions are concave up. We may use this to approximate $e = (e^{0.1})^{10} \approx 2.59$.