Exponential Functions

1. On his birthday last year, Sad invested $1000 dollars in a small company. This year he discovers that his investment has grown by 10% and is now worth $1100.

   Investment plan 1: the investment grows at a constant rate.

   Let $f(x)$ be the value of the investment x years after the initial investment.

   (a) Fill out the following table.

x	0	1	2	3
$f(x)$				

   (b) Write down a formula for $f(x)$.

   (c) By what percentage will the investment increase during the first year? During the second year? How about during the $(n+1)$-th year?

   Investment plan 2: the investment grows by 10% every year.

   Let $g(x)$ be the value of the investment x years after the initial investment.

   (d) Fill out the following table.

x	0	1	2	3
$g(x)$				

   (e) Write down a formula for $f(x)$.

   (f) By what percentage will the investment increase during the first year? During the second year? How about during the $(n+1)$-th year?

   Exponential functions are functions whose percent changes are constant!

2. Ernaya is doing an experiment of growing bacteria. (Not the strange ones that always grow in rectangular shape!) Suppose it begins with P_0 bacteria. Let $P(t)$ be the number of bacteria after t hours.

   (a) If the bacteria increase by 100% every 15 minutes, fill in the following table.

t	0	0.25	0.5	0.75	1
$P(t)$					

   (b) Write down a formula for $P(t)$ for each of the following cases:
   - the number of bacteria doubles every 15 minutes

   - the number of bacteria triples every hour

   - the number of bacteria triples every half an hour

   - the number of bacteria is halved every hour
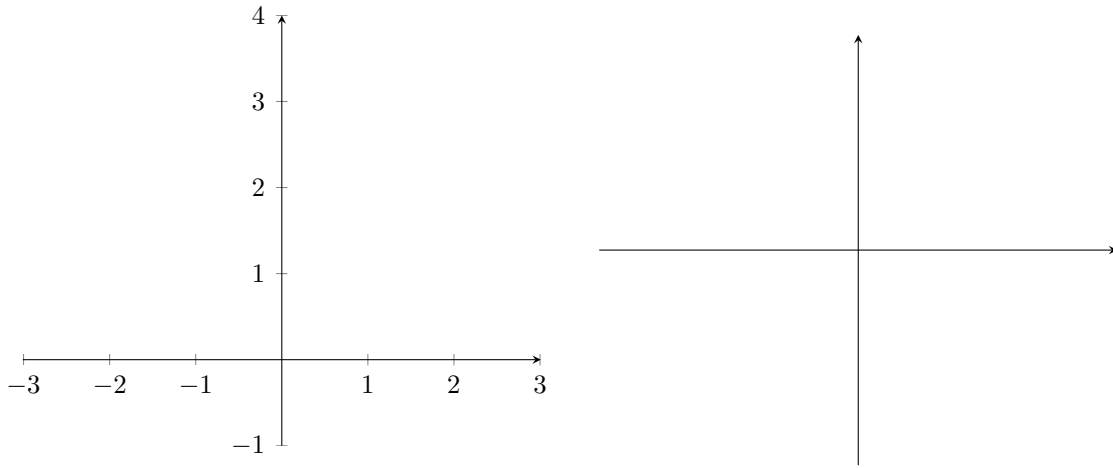
   - the number of bacteria is halved every 3 hours

3. Simplify the following expressions.
   (a) $\dfrac{[(x^5)^3 + \sqrt[3]{x^6} \cdot x^4]^2}{x^7}$

   (b) $\dfrac{(ab)^x}{b^{2x}}$

   (c) $\dfrac{2a^x + (2a)^x + a^{2x} + a^{x+2}}{a^x}$

4. Sketch the graphs of $y = 2^x$, $y = 3^x$, $y = (\frac{1}{2})^x$, $y = (\frac{1}{3})^x$ on the left, and sketch the graphs of their derivative on the right.

5. For each of the following function, find a sketch below which shows the general shape and position of the function's graph. (You may use a sketch more than once.)
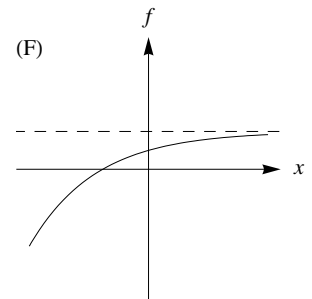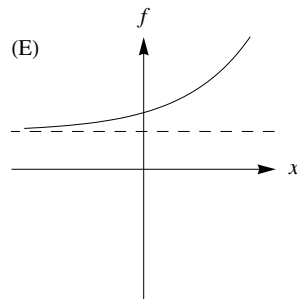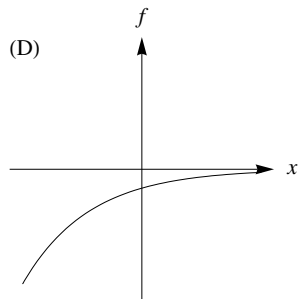
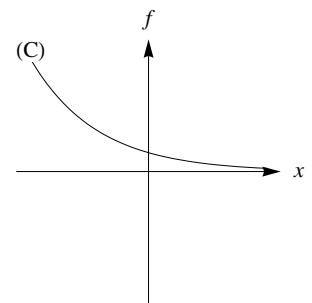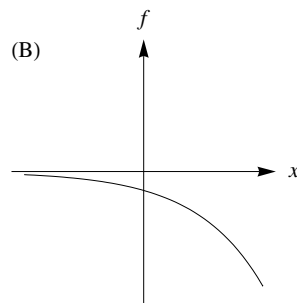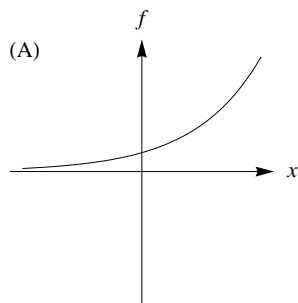(a) $f(x) = 5\left(\dfrac{1}{2}\right)^x$

(b) $f(x) = -1.5(0.06)^x$

(c) $f(x) = 10 + 1.5^x$

(d) $f(x) = 6\left(\dfrac{5}{4}\right)^x$

(e) $f(x) = 3^{-x}$

(f) $f(x) = 3 - 4^{-x}$

Exponential Functions – Solutions

1. (a)

x	0	1	2	3
$f(x)$	1000	1100	1200	1300

(b) $f(x) = 1000 + 100x$

(c) The percentage increase during the first year is 10%. The percentage increase during the second year is $\frac{1200-1100}{1100} \times 100\% \approx 9\%$ The percentage increase during the $(n+1)$-th year is

$$\frac{100}{1000 + 100n} \times 100\% = \frac{100}{10 + n}\%$$

which tends to 0 when n gets very large.

(d)

x	0	1	2	3
$g(x)$	1000	1100	1210	1331

(e) $g(x) = 1000 \cdot 1.1^x$.

(f) The percentage increase during the first year, second year, and $(n+1)$ year are all 10%.

2. (a)

t	0	0.25	0.5	0.75	1
$P(t)$	P_0	$2P_0$	$4P_0$	$8P_0$	$16P_0$

(b)
- $P_0 \cdot 16^t$
- $P_0 \cdot 3^t$
- $P_0 \cdot 9^t$
- $P_0 \cdot \left(\frac{1}{2}\right)^t$
- $P_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{3}}$

3. (a)

$$\frac{[(x^5)^3 + \sqrt[3]{x^6} \cdot x^4]^2}{x^7} = \frac{(x^{15} + x^2 \cdot x^4)^2}{x^7} = \frac{(x^{15} + x^6)^2}{x^7} = \frac{x^{30} + 2x^{21} + x^{12}}{x^7} = x^{23} + 2x^{14} + x^5$$
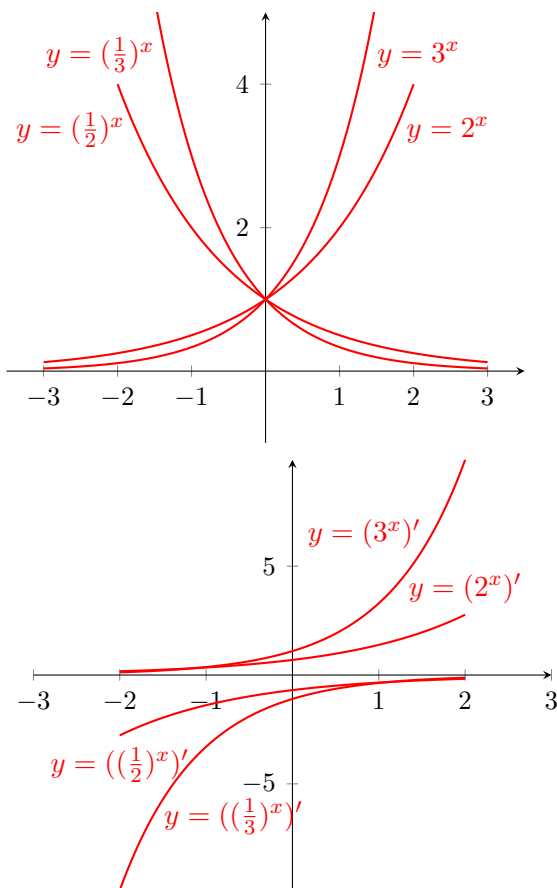
(b)

$$\frac{(ab)^x}{b^{2x}} = \frac{a^x b^x}{b^{2x}} = \frac{a^x}{b^x}$$

(c)

$$\frac{2a^x + (2a)^x + a^{2x} + a^{x+2}}{a^x} = \frac{2a^x + 2^x a^x + (a^x)^2 + a^x \cdot a^2}{a^x} = 2 + 2^x + a^x + a^2$$

4.

5. C.D.E.A.C.F.