Modeling and Interpreting Slope

1. Joey recently starts a part time job. It pays \$8 per hour for the first 6 hours he works each week, \$10 per hour for the second 6 hours, and \$12 per hour thereafter, up to a maximum of working 15 hours a week.

   (a) Write down the function $S(t)$ which describes Joey's salary if he works t hours a week.

   (b) Sketch the graph of $S(t)$.

   (c) Solve $S(t) = 100$. What does the equation mean?

   (d) Let $S'(t)$ be the function describing the slope of the graph of S. What is the practical meaning of $S'(t)$?

   (e) Sketch a graph of $S'(t)$. Be careful about the domain of $S'(t)$.

Look back to problem 3 on the worksheet of *Modeling with Functions* and answer the questions here in that setting.

2. Let $f(x) = \sqrt{x}$.

   (a) Sketch the graph of $f(x)$.

   (b) What is the average rate of change of f in the interval $[4, 9]$? How do you interpret this in the graph?

   (c) Find the secant line of f through the point whose x-coordinate is 4 and the point whose x-coordinate is 9.

   (d) Use the secant line in (c) to estimate $\sqrt{5}$. Is the estimate too high or too low?

   (e) Use the secant line in (c) to estimate $\sqrt{3}$. Is the estimate too high or too low?

(f) Find the secant line of f through the point whose x-coordinate is 4 and the point whose x-coordinate is $4 + h$.

(g) Let $h = 0.41$. Use the secant line from (f) to approximate $\sqrt{5}$. Is the estimate too high or too low?

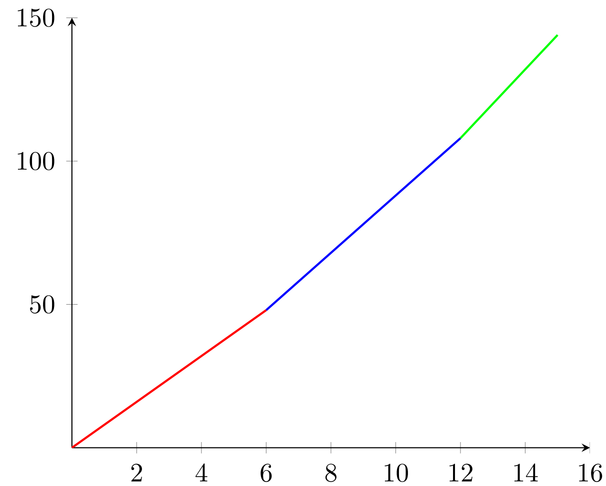
(h) Let $h = 0.0401$. Use the secant line from (f) to approximate $\sqrt{5}$. Is the estimate too high or too low?

Modeling and Interpreting Slope – Solutions

1. (a)

$$S(t) = \begin{cases} 8t & 0 \le t \le 6 \\ 10(t-6)+48 & 6 \le t \le 12, \\ 12(t-12)+108 & 12 \le t \le 15 \end{cases}$$

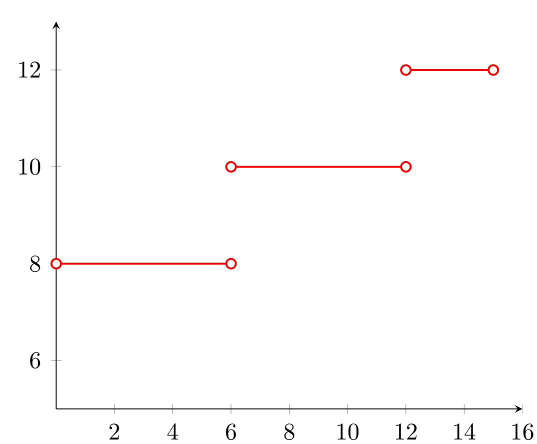
(b)



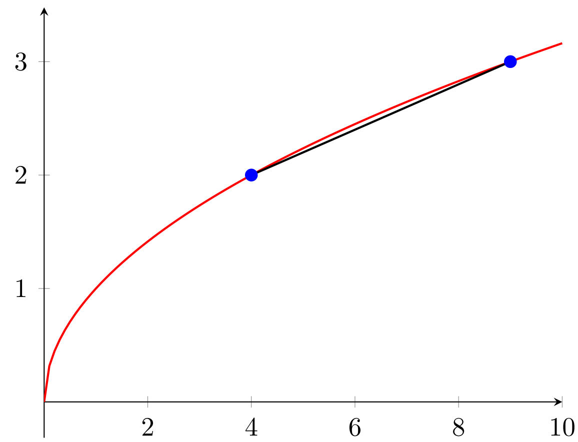
(c) For $0 \le t \le 6$, the range of $S(t)$ is $[0, 48]$, so we do not use the first case of $S(t)$ to solve $S(t) = 100$. For $6 \le t \le 12$, the range is $[48, 108]$, so we solve $S(t) = 10(t-6)+48 = 100$. The solution is $t = 11.2$ hours.
The equation means that $t = 11.2$ is the time Joey needs to work in a week in order to get \$100 dollars.

(d) $S'(t)$ describes the salary rate (dollars per hour) when Joey has worked t hours in the week.

(e)



2. (a)



(b) The average rate of change of f in $[4, 9]$ is the same as the slope of the secant line connection the point with x-coordinate 4 and the point with x-coordinate 9 on the graph. Hence it should be

$$\frac{3-2}{9-4} = \frac{1}{5}$$

(c) The slope of the secant line, which we have found above, is $\frac{1}{5}$. We use point-slope formula to write the equation of the secant line:

$$y - 2 = \frac{1}{5}(x - 4)$$

(d) Using the secant line $y = \frac{1}{5}(x - 4) + 2$, we approximate $\sqrt{5}$ as

$$\frac{1}{5}(5 - 4) + 2 = 2.2$$

This is an underestimate since the graph of $y = \sqrt{x}$ is concave down; in particular the secant line lies below the graph between the endpoints. In fact $\sqrt{5} \approx 2.236$.

(e) Using the secant line $y = \frac{1}{5}(x - 4) + 2$, we approximate $\sqrt{3}$ as

$$\frac{1}{5}(3 - 4) + 2 = 1.8$$

This is an overestimate since the graph of $y = \sqrt{x}$ is concave down; in particular the secant line lies above the graph outside the endpoints. In fact $\sqrt{3} \approx 1.732$.

(f) The slope of the secant line is

$$\frac{\sqrt{4+h} - 2}{(4+h) - 4} = \frac{(\sqrt{4+h} - 2)(\sqrt{4+h} + 2)}{h(\sqrt{4+h} + 2)} = \frac{h}{h(\sqrt{4+h} + 2)} = \frac{1}{\sqrt{4+h} + 2}$$

Using slope-point formula, the equation of the secant line is

$$y = \frac{1}{\sqrt{4+h}+2}(x-4)+2$$

(g) Plugging in $x = 5$ into the equation of the secant line we get

$$\frac{1}{\sqrt{4+0.41}+2}(5-4)+2 = \frac{1}{2.1}+2 \approx 2.476$$

This is an overestimate.

(h) Plugging in $x = 5$ into the equation of the secant line we get

$$\frac{1}{\sqrt{4+0.0401}+2}(5-4)+2 = \frac{1}{2.01}+2 \approx 2.496$$

This is an overestimate.