More on Functions

1. This is a continuation of the Question 6 on your problem set:

   Some friends are taking a long car trip. They are traveling east on Route 66 from Flagstaff, Arizona, through New Mexico and Texas and into Oklahoma.

   - Let f be the function that gives the number of miles traveled t hours into the trip, where $t = 0$ denotes the beginning of the trip.
   - Let g be the function that gives the car's speed t hours into the trip, where $t = 0$ denotes the beginning of the trip.

   Suppose they pass a sign that reads "entering Gallup, New Mexico," h hours into the trip.

   Write the following expressions using functional notation.

   (a) The car's speed 2 hours before entering Gallup.

   (b) The car's average speed in the first 3 hours of the trip.

   (c) The car's average speed in the second 3 hours of the trip.

   (d) The car's average speed in the half an hour after getting to Gallup, New Mexico.

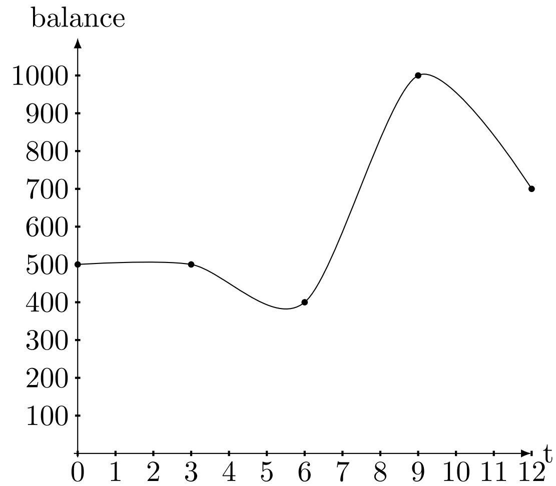
   Now let's do the contrary: interpret the following functional notation in words.

   (e) $g(h)$

   (f) $\dfrac{f(h+5) - f(h)}{5}$

   (g) $\dfrac{g(h+0.5) - g(h)}{0.5}$

2. The following is the graph function $f(t)$, which modles the balance in Chi-Yun's checking account t months after Sepetember 2015.



(a) What is the change of the balance between Sep. 2015 and Sep. 2016?

(b) What is the average rate of change of balance between Mar. 2016 and June 2016? What is the unit?

(c) How do you express the average rate of change of balance t months after Sep. 2015 in functional notation?

(d) What is the percentage change of balance between Dec. 2015 and March 2016?

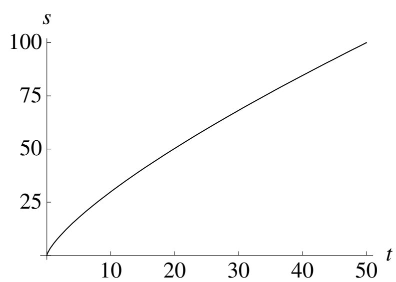
(e) How do you express the percentage change of balance t months after Sep. 2015 in functional notation?

Observation

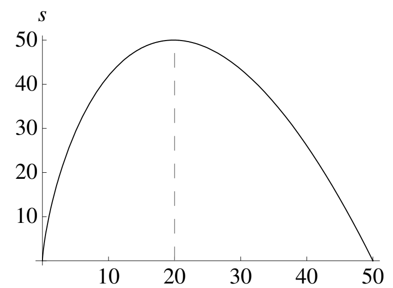
- For a concave up graph, a secant line is __________ the graph in between the endpoints of the secant line, and __________ outside.
- For a concave down graph, a secant line is __________ the graph in between the endpoints of the secant line, and __________ outside.

3. Mikaela is swimming a 100 m long race (one lap in a 50 m long pool). Let $s(t)$ be the distance from the starting position t seconds after the race starts.

   (a) Which of the following is a more reasonable graph for $s(t)$? Why? What should be the meaning of the other graph?



(A)



(B)

   (b) According to the graph you chose, how long did it take Mikaela to finish the race?

   (c) What was her average velocity over the first 20 seconds of the race?

   (d) What was her average velocity over the last 50 m of the race?

Definition

- Velocity:
- Speed:

4. Suppose $f(x)$ is a linear function defined on $(-\infty, \infty)$. How does the average rate of change of f on $[0, 5]$ compare to the average rate of change of f on $[0, 500]$?

More about Functions – Solutions

1. (a) $g(h-2)$.
   (b) $\dfrac{f(3)-f(0)}{3-0} = \dfrac{f(3)}{3}$.
   (c) $\dfrac{f(6)-f(3)}{6-3} = \dfrac{f(6)-f(3)}{3}$.
   (d) $\dfrac{f(h+0.5)-f(h)}{0.5}$.
   (e) The car's speed when reaching Gallup.
   (f) The average speed in 5 hours after getting to Gallup.
   (g) The average acceleration in half an hour after getting to Gallup.

2. (a) $700 - 500 = 200$ dollars.
   (b) $\dfrac{1000-400}{3} = 200$ dollars/month.
   (c) $\dfrac{f(t)-f(0)}{t-0} = \dfrac{f(t)-500}{t}$ dollars/month.
   (d) $\dfrac{400-500}{500} \times 100\% = -20\%$.
   (e) $\dfrac{f(t)-f(0)}{f(0)} \times 100\% = \dfrac{f(t)-500}{5}\%$.

3. (a) Choice (B) is more reasonable: when Mikaela reaches the opposite end of the pool, she is 50 m from the starting point. At the end of the race, she is back at the starting point. (A) should represent the function of swimming distance at time t.
   (b) The amount of time it takes to finish the race is the time when the position is 0 again. Hence it is 50 seconds.
   (c) $\dfrac{s(20)-s(0)}{20-0} = \dfrac{50}{20} = 2.5$ m/s.
   (d) $\dfrac{0-50}{50-20} = -\dfrac{5}{3}$ m/s.

4. They are the same. The average rate of change of a linear function is constant, which is the slope of its graph: a line.